On the Data Fusion of Accelerometer and Gyroscope Sensors Using Complementary Filter

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Abstract

With small sizes and lower cost, accelerometers and gyroscopes in the form of MEMS unit have been widely used in many diverse areas of applications involving detections of motion, vibration, attitude of various airborne objects, and health monitoring. Each sensor has systematic and parasitic errors that affect its output accuracy, and thus limit its functioning when it is used individually. Both sensors may be fabricated on the same silicon die known as the inertial measurement unit (IMU). When put to use together in an application, appropriate data fusion technique is an issue that must be considered for more efficient implementation. This paper aims to 1) present a unified concept of complementary filters together with outline of IMU data fusion sequence followed by the technique used in a classical Kalman filter as a comparison; 2) present an example which demonstrates the overall performance of both filter types. By using block diagrams with a flow coding of data processing sequence, the operation by the complementary filter can be compared and contrasted with that of Kalman filter more easily. A simple example from a separate source demonstrates that a complementary filter can offer an overall output result comparable to that of Kalman filter with an additional speed and simplicity advantage. In this case all the filter gains are constant. The general structures of the complementary filter and the Kalman filter have been shown and their performances are demonstrated. It is noted that in the low-noise case, the complementary filter can simply be a weighted-average, constant-gain filter, but for high noise variances, a more suitable low-pass and high-pass filter design should be considered.

Keywords: *accelerometer, gyroscope, IMU, data fusion, complementary filter, Kalman filter*

บทคัดย่อ

ด้วยคุณสมบัติที่มีขนาดเล็กและราคาต่ำทำให้ตัววัดความเร่งและไจโรสโคปที่ผลิตด้วยเทคโนโลยีระบบเครื่องกลไฟฟ้าจุลภาคถูกนำมาใช้กันอย่างกว้างขวางในงานประเภทที่แตกต่างกันอย่างหลากหลายในส่วนที่เกี่ยวกับการตรวจจับการเคลื่อนไหว การสั่นสะเทือน ตำแหน่งและทิศทางการเคลื่อนที่ของวัตถุในอากาศและการตรวจวัดสุขภาพ เซนเซอร์แต่ละชนิดจะมีค่าความผิดพลาดเนื่องจากระบบโครงสร้างของเซนเซอร์เองและจากอิทธิพลภายนอกทำให้มีผลต่อค่าความละเอียดของเอาท์พุทที่วัดซึ่งเป็นการจำกัดขอบเขตการใช้งานเฉพาะตัวของเซนเซอร์นั้น เซนเซอร์ทั้งคู่อาจถูกผลิตอยู่บนแผ่นวงจรรวมซิลิคอนชิ้นเดียวกันและเรียกรวมว่า ตัววัดแรงเฉื่อย เมื่อถูกใช้งานร่วมกันเทคนิคที่เหมาะสมในการหลอมรวมข้อมูลจะเป็นประเด็นที่ต้องมีการพิจารณาเพื่อให้การใช้งานนั้นมีประสิทธิภาพมากยิ่งขึ้น บทความนี้มีเป้าหมายเพื่อ 1) นำเสนอแนวคิดรวมของฟิลเตอร์เติมเต็มพร้อมด้วยรูปแบบแสดงขั้นตอนการหลอมรวมข้อมูลของตัววัดแรงเฉื่อยรวมทั้งแสดงเทคนิคที่ใช้ในกรณีของฟิลเตอร์คาลมานแบบดั้งเดิมเพื่อการเปรียบเทียบ 2) นำเสนอตัวอย่างให้เห็นถึงผลการทำงานของฟิลเตอร์ทั้งสองประเภท โดยการใช้แผนภาพบล็อกและภาพแสดงการไหลของข้อมูลผ่านขบวนการอย่างเป็นขั้นตอน ทำให้สามารถเปรียบเทียบการทำงานและเห็นความเหมือนและความแตกต่างของฟิลเตอร์เติมเต็มจากฟิลเตอร์คาลมานได้ง่ายขึ้น ผลการวิจัยแสดงให้เห็นว่าเอาท์พุทจากฟิลเตอร์เติมเต็มสามารถให้เอาท์พุทโดยรวมคล้ายคลึงกับเอาท์พุทของฟิลเตอร์คาลมานโดยมีข้อดีในด้านความเร็วและความง่ายในการนำหลักการมาใช้และในกรณีตัวอย่างนี้อัตราขยายของฟิลเตอร์มีค่าคงที่ โดยสรุปได้แสดงโครงสร้างทั่วไปของฟิลเตอร์เติมเต็มและฟิลเตอร์คาลมานและผลการทำงานที่มีลักษณะใกล้เคียงกัน ในกรณีที่มีสัญญาณรบกวนต่ำฟิลเตอร์เติมเต็มอาจเป็นเพียงฟิลเตอร์ที่ใช้อัตราขยายเป็นแบบเฉลี่ยถ่วงน้ำหนักและค่าคงที่ แต่อย่างไรก็ดีเมื่อระดับของสัญญาณรบกวนมีค่าสูงขึ้นก็ควรพิจารณาการออกแบบฟิลเตอร์เติมเต็มให้มีคุณสมบัติกรองความถี่ต่ำและสูงที่เหมาะสม

คำสำคัญ: *ตัววัดความเร่ง ไจโรสโคป ตัววัดแรงเฉื่อย การหลอมรวมข้อมูล ฟิลเตอร์เติมเต็ม ฟิลเตอร์คาลมาน*

1. Introduction

Accelerometer and gyroscope are two instruments that are used to measure acceleration and angular velocity. Alternatively that means the measurement of position, orientation, and velocity of moving objects. With the improvement of micro-electro-mechanical systems (MEMS) technology, the two instruments have been fabricated into sensors residing in a single, miniaturized silicon substrate. These two sensors form the basic inertial measurement unit (IMU) which may include other sensors such as magnetometers. With its low cost and small size, many functions that an IMU offers have found many applications such as gesture commands for applications and control in smartphones, enhanced gaming, augmented reality, pedestrian and vehicle navigation, health and fitness monitoring, to name a few. An example of a MEMS-based IMU that combines a 3-axis gyroscope, a 3-axis accelerometer, and additional features such as temperature sensor and Digital Motion Processor™ (DMP) in a single 4x4x0.9mm package is MPU-6050 from InvenSense as shown in Figure 1.

MPU-6050 IMU chip

Figure 1 MPU-6050 MotionTracking™ device in a 4x4x0.9mm package (courtesy of InvenSense and Sparkfun Electronics, USA).

Due to the fabrication constraints, the MEMS-based IMU is subjected to various errors and usually needs calibration before use. For the IMU consisting of tri-axis accelerometer and tri-axis gyroscope, the two sensors may be modelled as

$$^{s}\boldsymbol{a} = \mathbf{K}_a\tilde{\boldsymbol{a}} + \mathbf{b}_a + \boldsymbol{v}_a \quad (1a)$$

$$^{s}\boldsymbol{\omega} = \mathbf{K}_g\tilde{\boldsymbol{\omega}} + \mathbf{b}_g + \boldsymbol{v}_g \quad (1b)$$

where $^{s}\boldsymbol{a}$, $^{s}\boldsymbol{\omega}$ are the measured vectors of the tri-axis accelerometer sensor and the tri-axis gyroscope sensor respectively; $\tilde{\boldsymbol{a}}$ is the 3×1 vector of total acceleration, consisting of the gravity and the acceleration due to sensor-body movement; $\tilde{\boldsymbol{\omega}}$ is the true angular velocity 3×1 vector of the sensor. $\mathbf{K}_a$, $\mathbf{K}_g$ are the diagonal scale-factor matrices. $\mathbf{b}_a$ and $\mathbf{b}_g$ are bias vectors; $\boldsymbol{v}_a$ and $\boldsymbol{v}_g$ are assumed uncorrelated white Gaussian measurement noise with zero means. The above equation is a simplified model which does not take into account the cross-axis sensitivity, cross-coupling, and misalignment (Sabatini, 2006).

The accelerometer gives the *x-y-z* acceleration component values which basically can be used to calculate inclination angles, and therefore it may be initially considered sufficient. However the acceleration is caused by both gravitation and by movement of the device. As a result, even if the accelerometer is in a relatively stable state, it is still very sensitive to vibration and mechanical noise in general. The gyroscope measures the rate of change of angle along the *x-y-z* axes and can be used to smooth out these accelerometer errors. However gyroscopes have drift which will cause large error even after being used for a short period. Nevertheless by properly combining data that comes from both accelerometer and gyroscope, one can obtain a relatively better estimate of current device inclination than what one would obtain by using the accelerometer data alone.

The literature on fusing accelerometer and gyroscope data is abundant (OlliW, 2013). Data may be fused using simply weighted average, or by complementary filter (Colton, 2007); the latter can have many extensions (Mahony et al., 2005), (Mahony et al., 2008), (Euston et al., 2008). Linear Kalman filter (KF) and their extended version (EKF) is more traditional and well established, but is usually more complicated and requires high computational effort (Sabatini, 2006). Mahony filter (Mahony et al., 2008), and Madgwick filter (Madgwick et al., 2011) are two separately introduced filters with similar results. The latter two are recently gaining more interest among hobbyists. Comparisons among these filters have been attempted in various aspects and many similarities among them have been brought up (Higgins, 1975), (Giacomel, 2011). It was also pointed out that the complementary filter and Kalman filter in basic case can lead to identical updated equations (OlliW, 2013).

In the following part of this paper the basic concept of data fusion will be described and its implementation based on data obtained from the commercially available chips, i.e., ADSL 335 triple-axis accelerometer and a compatible gyroscope will be given.

2. Objectives

The objectives of the paper are: firstly, to present a clear concept of data fusion for the IMU (consisting of accelerometer and gyroscope) and simple analysis of a complementary filter together with a theoretical comparison with an established Kalman filter. Secondly, to present concisely both in a generic script and a chart form the software implementation of a complementary filter at the chip-interface level based on a low-cost and generally available IMU. Thirdly, to present a brief, comparative result demonstrating the performance of both types of filters for an angle measurement. Conclusions and suggestions are finally given.

3. Materials and methods

3.1 Complementary filter

The basic idea of complementary filter is shown in Figure 2 below where x and y are noisy measurements of some signal z and $\hat{z}$ is the estimate of z produced by the filter.

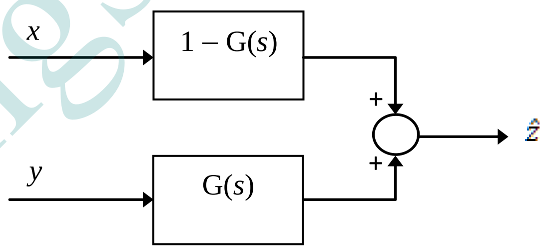

Figure 2 Basic complementary filter.

If G(s) is a low-pass filter, then 1−G(s) is a high-pass filter. This is the basic form (Higgins, 1975) which can be implemented differently depending on G(s) and how it is discretized. In a slightly simplified version of the complimentary filter, the data from gyro and accelerometer are combined simply by the weighted average concept without regard to the type of noises appeared in the data from the two sensors. This is shown in the following block diagram.

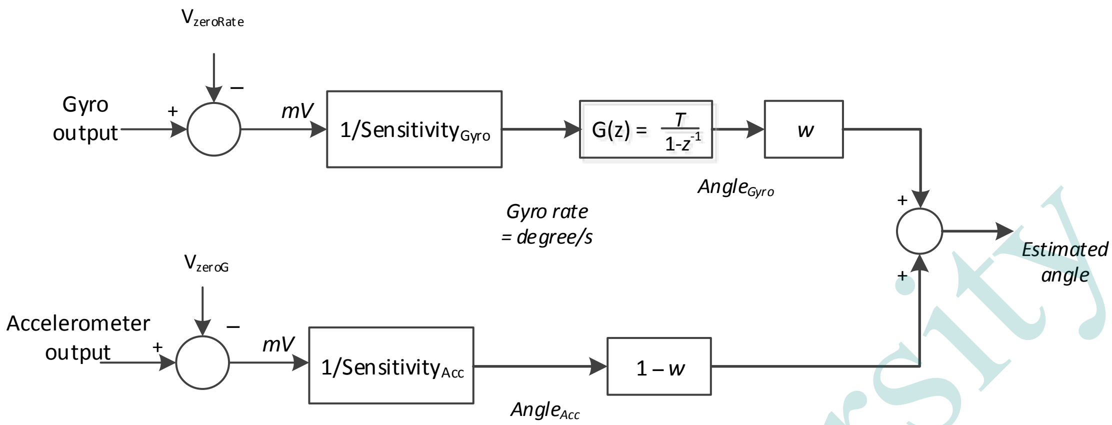

Figure 3 Block diagram of the basic complementary with indicated values and units.

The calculation steps involved are explained based on the ADSL335 3-axis accelerometer and a compatible gyroscope as follows:

Let Racc = [RxAcc, RyAcc, RzAcc] be the inertial force vector as measured by accelerometer that consists of the following components:

RxAcc = (AdcRx * Vref/1023 – VzeroG)/Sensitivity (2a)

RyAcc = (AdcRy * Vref/1023 – VzeroG)/Sensitivity (2b)

RzAcc = (AdcRz * Vref/1023 – VzeroG)/Sensitivity (2c)

where

AdcRx, AdcRy, AdcRz = values read from three channels of the ADC module assuming to have 10 bits.

Vref = reference voltage of the ADC module, assuming it is 3.3V

VzeroG = voltage corresponding to 0g, assuming it is 1.65V

Sensitivity = accelerometer sensitivity, usually expressed in mV/g (= 478.5)

Note that

|Racc| = SQRT(RxAcc^2 + RyAcc^2 + RzAcc^2) (3)

In the normalized form,

Racc(normalized) = 1 = [RxAcc/|Racc|, RyAcc/|Racc|, RzAcc/|Racc|] (4)

Let Racc = current reading from accelerometer
Rgyro = value obtained from Rest(n-1) and current gyro readings
Hence
Rest(n) = (Racc*w1 + Rgyro*w2)/(w1+w2) (5)

Dividing both numerator and denominator of the fraction by w1,

Rest(n) = (Racc*w1/w1 + Rgyro*w2/w1)/(w1/w1+w2/w1)

= (Racc + Rgyro*wgyro)/(1 + wgyro) (6)

Hence

RxEst(n) = (RxAcc + RxGyro*wGyro)/(1 + wGyro) (7a)

RyEst(n) = (RyAcc + RyGyro*wGyro)/(1 + wGyro) (7b)

RzEst(n) = (RzAcc + RzGyro*wGyro)/(1 + wGyro) (7c)

Let these be normalized again:

R = SQRT(RxEst(n)^2 + RyEst(n)^2 + RzEst(n)^2) (8)

RxEst(n) = RxEst(n)/R (9a)

RyEst(n) = RyEst(n)/R (9b)

RzEst(n) = RzEst(n)/R (9c)

Then the angle of inclination can be found from

CosX =cos(Axr) = RxEst(n) (10a)

CosY =cos(Ayr) = RyEst(n) (10b)

CosZ =cos(Azr) = RzEst(n) (10c)

Notice that the two weights w1 and w2 are implemented, and this is basically chosen on an experimental basis. The data flow and computational steps are visualized in the following Figure 4.

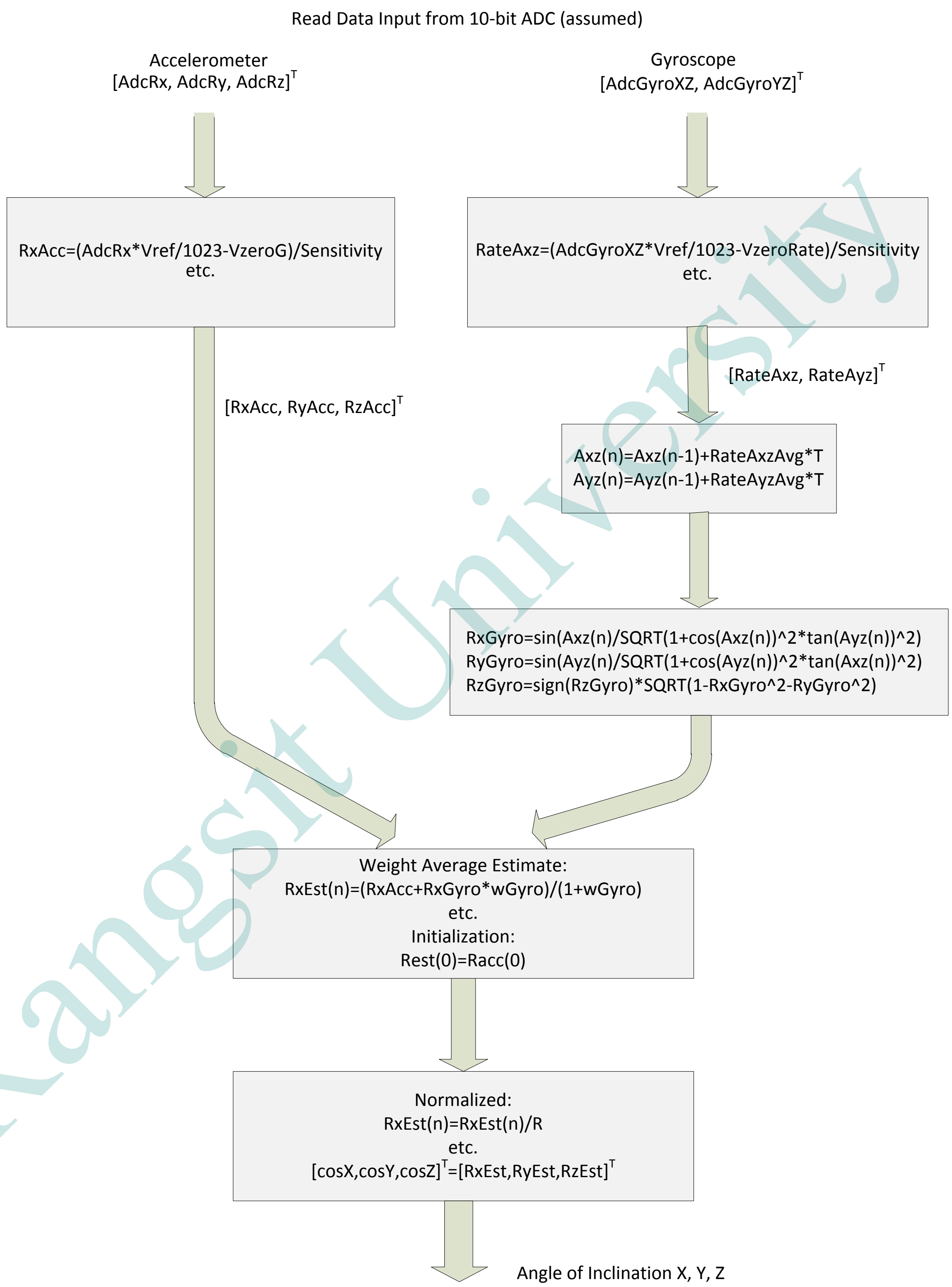

Figure 4 Data flow and computational steps for accelerometer and gyro data fusion.

3.2 Kalman filter

The Kalman filter that is implemented on the similar platform can be displayed in the following block diagram in Figure 5.

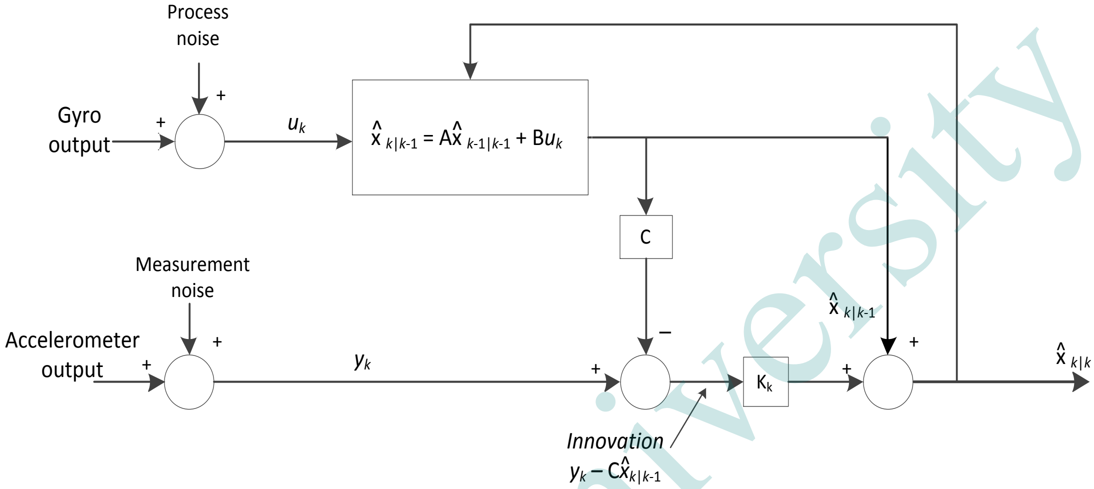

Figure 5 IMU Data fusion using Kalman filter.

The sensor model for each axis is proposed as follows:

$$x_{k+1} = Ax_k + Bu_k \tag{11a}$$
$$y_k = Cx_k + v_k \tag{11b}$$

which can be expressed as

$$\begin{bmatrix} alpha \\ bias \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & dt \\ 0 & 1 \end{bmatrix}\begin{bmatrix} alpha \\ bias \end{bmatrix}_k + \begin{bmatrix} dt \\ 0 \end{bmatrix} u_k \tag{12a}$$

$$y_k = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} alpha \\ bias \end{bmatrix} + v_k \tag{12b}$$

It is shown here that Kalman filter has u_k (the measured gyro output in degree/s) and y_k (the measured accelerometer output in degree) as filter inputs, and the estimated state $\hat{x}_{k|k}$ as its output in degree. Noises are embedded in u_k with noise covariances and unit conversions are to be taken care of separately and appropriately. The above equations are to be dimensionally extended to all three axes belonging to each sensor.

4. Results and Discussion

It can be seen that the computation work load and level of complexity of the two filters were quite different. For complementary filter, two weights, w1 and w2 were required for adjustment. For Kalman filter, process and measurement noise covariances, including second moment of states were all to be estimated and experimented with. Few assumptions were made that they were constant and the modelling was accurate. For comparative study of the two filter performances, a separate result (Giacomel, 2016) is presented here as shown in Figure 6. The values shown are the angles between two axes x and y in a single plane when the rotation was made around z-axis of the two sensors that were aligned.

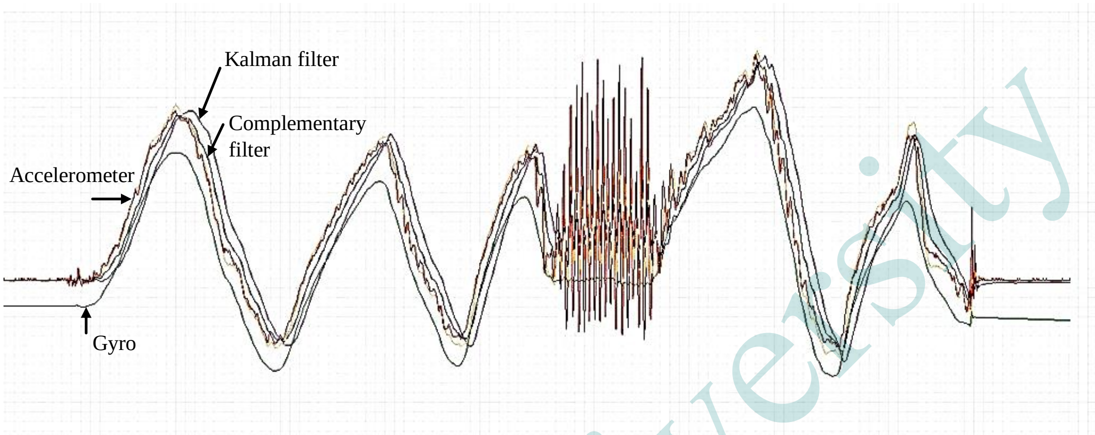

Figure 6 A performance comparison of the complementary filter and Kalman filter.

It can be observed that in the overall, both filters behave very similarly, with Kalman filter responses slightly slower than complementary filter. The result also shows that the accelerometer is very sensitive to the vibrations that occurs near the middle of the experiment whereas the gyrocope is very immune to such disturbance. These observations are in line with others that were reported in literatures.

6. Conclusion

The general structures of the complementary filter and the Kalman filter have been shown and the performances of both filters were demonstrated. In the case as demonstrated in Figure 3 only a constant gain was considered, i.e., $G(s) = w$ and this should be sufficient in low-noise case. However if the noise in the relevant signal is mostly high frequency, then $G(s)$ is to be made as a low-pass filter to filter out high-frequency noise and then $1-G(s)$ is the complement, i.e., a high-pass filter (Higgins, 1975). In such case the filter design may become an issue that leads to complicated situation similar to that in the implementation of Kalman filter where the noise statistics and system modeling are to be considered more elaborately. Since complementary filters can be configured differently from what has been described, further improvement may be possible and that will probably depends on intuitive knowledge of each individual application.

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