Design and Development of Lesson Plans on Elementary Counting Techniques Through The Integration of the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers

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Abstract

This article aims to design and develop lesson plans by integrating the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers. The lesson plan design incorporates the DAPIC Problem-Solving Process along with the Four Corners and Diamond Graphic Organizers, consisting of five steps: Step 1 Define, Step 2 Assess, Step 3 Plan, Step 4 Implement, and Step 5 Communicate, Exchange, and Summary. The developed lesson plans cover two topics: (1) The Elementary Counting Techniques (the Addition and Multiplication Principles) and (2) Linear Permutations (of Distinct and Indistinct Objects). The quality of the lesson plans was assessed by experts, and their feedback was used for revisions before the implementation. The assessment results, based on a Likert scale, indicated an "Excellent" level of quality. This suggests that the lesson plans can be beneficial for mathematics teachers in applying them to classroom instruction to enhance students' mathematical problem-solving skills, particularly for the high school students.

Keywords**:** *The DAPIC Problem-Solving Process, The Four Corners and Diamond Graphic Organizers, Problem-Solving Skills*

1. Introduction

A report on the results of the Programmed for International Student Assessment (PISA) 2018 and the National Basic Education Examination (O-NET) for Grade 9 in Thailand in the academic year 2021 showed that there are still big problems with how math is taught and learned. These deficiencies highlight the urgent need for educators to implement more effective strategies and methodologies in their teaching practices. Students can enhance their mathematical skills and improve their overall academic performance by fostering a deeper understanding of problem-solving techniques. The PISA and O-NET exams emphasize complex and diverse mathematical problem-solving skills. Most students are proficient only in basic arithmetic operations such as addition, subtraction, multiplication, and division. However, when presented with problem-solving exercises, many students struggle to solve them effectively (NIETS, 2021; IPST, 2021). As a result, students cannot apply their mathematical knowledge, skills, and processes to solve mathematical problems (Suwannatrai & Thongmoon, 2020). Therefore, teachers should emphasize effective teaching strategies and resources to develop students' problem-solving skills.

The DAPIC Problem-Solving Process is a mathematical problem-solving method developed for use in the integrated curriculum of mathematics, science, and technology by the Center for Mathematics, Science, and Technology at the University of Illinois. The purpose of this process is to develop problem-solving skills, train analytical and problem-solving planning systematically, carry out problem-solving activities, summarize answers logically, and help promote skills and the ability to solve mathematics problems. It is a flexible and uncomplicated problem-solving process (Boonraksa, 2018; Wongjansau, 2012). Additionally, it has been found that students are able to solve problems effectively by applying the knowledge, skills, and principles they have learned in class. Furthermore, they can adapt these methods to suit real-life situations and challenges, resulting in efficient problem-solving and optimal outcomes (Bunyuen, Chuntra, & Somchaipeng, 2023),

Another problem-solving process that incorporates a graphical diagram is the Four Corners and Diamond Graphic Organizers, which emerged from the foundational work of Gould and Gould (Potong & Chanchusakun, 2022). who developed the Four-Square Writing method to enhance teaching strategies and improve students' fundamental writing skills in descriptive, explanatory, and persuasive genres. Zollman (2009) was developed by someone who expanded the framework, recognizing that human thought processes are inherently diverse and nonlinear. He noted that learners often struggle to start problem-solving and encounter difficulties due to limited procedural strategies (Zollman, 2009;2012). To address these challenges, the Four Corners and Diamond Graphic Organizers were adapted as problem-solving tools, particularly in mathematics education. Limond (2012) further emphasizes that many students face confusion when tackling mathematical problems, reinforcing the need for tools that help systematic brainstorming and structured problem-solving. Sai, Shahrill, Tan, and Han (2018) show how the visual and graphic elements within the graphic organizer can show the connections between statements, conditions, concepts, and ideas within the learning activity. It can also break down the problem into small parts.

Studying the DAPIC Problem-Solving Process reveals that many students struggle with initiating and systematically progressing through problem-solving tasks (Lampai, 2019). Specifically, difficulties often arise during the problem identification stage (Define), where students do not understand or effectively analyze the problem. Furthermore, students may meet challenges in planning and implementing (Implement) a solution, either due to a lack of structured planning or the incorrect sequencing of steps, preventing them from arriving at correct results (Boonraksa, 2018; Widodo, Nayazik, & Prahmana, 2019). Research also shows that some students neglect to document known information from the problem or do not record it accurately during the Define stage (Wijiya, 2020). These limitations highlight the need for tools that enhance students' ability to organize and process mathematical information effectively. In this context, the Four Corners and Diamond Graphic Organizers are complementary frameworks to strengthen the DAPIC Problem-Solving Process. This flexible and adaptable approach emphasizes understanding the problem and planning solutions in a way that aligns with and reinforces students' mathematical knowledge (Gunasegar & Maat, 2022).

2. Objectives

The objective of this research is to design and develop lesson plans by integrating the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers.

3. Materials and Methods

3.1 Methodology:

This section outlines the process of designing lesson plans and the tools used to assess their quality, specifically lesson plans and assessment forms. The process of designing the lesson plans is divided into three steps:

1) Designing the Lesson Plans: Developing the initial lesson plan structure.

2) Quality Assessment: Conducting a review of the lesson plans by three experts.

3) Revision and Improvement: Revising or refining the lesson plans based on feedback and suggestions provided by the experts.

The three experts involved in the quality assessment process possess the following qualifications:

1) Expert 1: A secondary school teacher with the professional rank of "Senior Professional-Level Teacher in Thailand (K3)," specializing in mathematics, and more than 10 years of teaching experience.

2) Expert 2: A secondary school teacher with the professional rank of "Professional-Level Teacher in Thailand (K2)," specializing in mathematics, and more than 10 years of teaching experience.

3) Expert 3: A university lecturer specializing in mathematics, with over 10 years of teaching experience in higher education.

3.2 Materials:

This section describes the tools used in the study, including the lesson plan quality assessment form and the lesson plans developed using the DAPIC Problem-Solving Process integrated with the Four Corners and Diamond Graphic Organizers.

3. 2. 1 The quality assessment form for the lesson plans: This form was designed to receive quality assessments for lesson plans by experts. Additionally, the quality assessment form for the lesson plans is presented in Table 1.

Table 1 The assessment form for assessing the quality of the lesson plans

Assessment List	Quality Level				
	5	4	3	2	1
1. Indicator/Objectives					
1.1 The correctness of the principles of writing objectives.					
1.2 The consistency of learning objectives with learning activities.					
1.3 The consistency of objectives that can be measured and evaluated.					
2. Content					
2.1 The correctness of the language used is clear and easy to understand.					
2.2 The consistency of the content with the expected learning outcomes.					
2.3 The consistency of the content with the learning activities.					
3. Content/Learning content					
3.1 The content is accurate, clear, and easy to understand.					
3.2 The consistency of learning content with indicators/learning objectives.					
3.3 The consistency of learning content with learning activities.					
4. Learning Management Process					
4.1 Learning activities are consistent with the learning activity organizing process by integrating the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers.					
4.2 Learning activity helps students understand and identify problems (Define).					
4.3 Learning activity helps students evaluate problems to select mathematical knowledge to use in solving problems (Assess).					
4.4 Students can brainstorm to plan a solution (Plan) and write a sequence of steps to solve problems.					
4.5 Students can implement the plan by using mathematical knowledge, principles, or concepts (Implement).					
4.6 Students can write a summary of the results of the problem, including the reasons for obtaining the answer, and exchange the results with others (Communicate, Exchange, and Summary).					
4.7 This activity encourages students to communicate the solution clearly and confidently.					
4.8 Group work through activities by integrating the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers helps develop teamwork skills and exchange ideas.					
4.9 Learning activities are appropriate for the given time.					
5. Learning Media					
5.1 The learning media used are clear, easy to understand, and interesting.					
5.2 The learning media used are consistent with the content and learning activities.					
5.3 The DAPIC Problem-Solving Process diagram in conjunction with the Four Corners and Diamond Graphic Organizers helps promote the organization of students' thinking.					
6. Measurement and Evaluation					
6.1 The consistency of measurement and evaluation with learning objectives.					
6.2 The consistency of measurement and evaluation with learning activities.					
6.3 Measurement tools (e.g., tests and assessments) are comprehensive and relevant to the learning content.					
6.4 The scoring rubric corresponds to all 5 steps of the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers.					
Total					
Average Score					

3.2.2 Lesson plan design: This section will include steps for integrating the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers as a tool to help develop high school students' mathematical problem-solving skills. It also studies various topics from the interesting mathematics course, based on the mathematics curriculum provided by the Institute for the Promotion of Teaching Science and Technology (IPST) for high school students; various topics are of interest. However, the lesson plans are designed around the Elementary Counting Technique and Linear Permutations. These topics were selected due to their potential for creating diverse problem-based scenarios that require critical thinking and analysis. Additionally, they aim to enhance students' mathematical problem-solving skills.

The lesson plan design is structured by integrating the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers. The lesson content is divided into four lesson plans, as follows:

1) The Elementary Counting Technique
    1.1) Addition Principle
    1.2) Multiplication Principle
2) Linear Permutation
    2.1) Linear Permutations of Distinct Objects
    2.2) Linear Permutations of Indistinct Objects

Each lesson plan includes activity sheets and exercises designed around problem-based scenarios related to the principles of the Element Counting Techniques and Linear Permutations, as outlined in the instructional topics. These activities are implemented by integrating the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers. This structured approach ensures students engage in critical thinking and systematic problem-solving throughout the learning process.

3.2.2.1 The DAPIC Problem – Solving Process

The DAPIC Problem-Solving Process was developed for use in the integrated mathematics, science, and technology curriculum of the Center for Mathematics, Science, and Technology (Illinois State Univ, 1998). It is based on Polya's Problem-Solving Process, the Model of Scientific Inquiry, and the Shewhart Cycle (Meier, Hovde, & Meier, 1996; Sumirattana, Makanong, & Thipkong, 2017; Wongjansau, 2012; Suwannapaeng & Kongson, 2023; Suwannatrai & Thongmoon, 2012).

The DAPIC Problem-Solving Process is designed to enhance skills and abilities in mathematical problem-solving. It is a flexible and straightforward method that can be adapted or modified, with some steps being optional. Additionally, the process allows for backtracking or restarting when encountering issues at any stage. The DAPIC Problem-Solving Process consists of five steps, as follows:

Step 1 Define: Understand the problem by identifying what needs to be determined, what is known, and what is required to solve the problem.

Step 2 Assess: Perform a problem-solving assessment by gathering relevant information or knowledge needed to solve the problem. This step involves connecting the collected information to the problem's requirements.

Step 3 Plan: Develop a plan or strategy to solve the problem. This includes identifying appropriate methods based on the nature of the problem and organizing the steps sequentially.

Step 4 Implement: Execute the planned solution to arrive at an answer. This step involves applying mathematical knowledge, principles, or concepts while also refining the action plan as necessary to address the problem effectively.

Step 5 Communication: Analyze and conclude the results. This step includes presenting the findings to others, exchanging insights, and evaluating the results to ensure accuracy and consistency, often through written worksheets.

From the study, the DAPIC Problem-Solving Process is a flexible and straightforward framework. Certain steps can be omitted, reversed, or revisited when encountering issues at any stage, depending on the nature of the problem. However, it was observed that students often struggle with solving mathematical problems due to difficulties in understanding or analyzing the problem, which indicates errors during the

problem definition stage. Additionally, some groups of students were unable to connect their mathematical knowledge to the problem. As a result, they struggled to formulate a coherent problem-solving sequence during the planning stage (Plan) and faced challenges in executing the solution during the implementation stage (Implement).

3.2.2.2 The Four Corners and Diamond Graphic Organizers

The Four Corners and Diamond Graphic Organizers focus on understanding and problem-solving planning in a format that connects students' existing knowledge with what they need to learn additionally (Gunasegar & Maat, 2022). Graphic organizers help students to differentiate and learn more effectively. It is a tool for solving mathematics problems that was developed following the Four-Square Writing method by Gould and Gould (Potong & Chanchusakun, 2022). Zollman (2009) developed it as a tool that enables students to solve mathematical problems by recording information in each section without needing to consider sequential steps. The Four Corners and Diamond Graphic Organizers are useful tools for many students to begin solving new problems and are a good resource for students who have more difficulty following math steps than others (Zollman, 2009;2012). Illinois State Univ (1998) found that most students were confused about solving problems. Therefore, they developed "The Four Corners and Diamond Graphic Organizers" as a tool for mathematical problem-solving.

The Four Corners and Diamond Graphic Organizers is versatile tools that support students in identifying connections within data. By presenting information through a structured graphic organizer, students can gain a clearer understanding of relationships within the data (Sai, Shahrill, Tan, & Han, 2018). This approach allows them to break down complex problems into smaller, more manageable components, thereby improving their comprehension of mathematical concepts (Sai et al., 2018). Additionally, it enhances efficiency in problem-solving. The organizer consists of the following five parts:

Part 1 Main Idea: Rewrite what the problem is asking for in your own words.

Part 2 Connection: Identify and write down key information from the problem. Relate it to real-life scenarios and consider whether the students have encountered or solved a similar situation before.

Part 3 Brainstorm: Develop and write potential strategies or mathematical methods for solving the problem.

Part 4 Solve: Perform the necessary calculations to solve the problem, following the steps outlined during brainstorming.

Part 5 Write: Summarize the solution step by step, explaining the reasoning behind each step to justify the answer.

From the study, the Four Corners and Diamond Graphic Organizers represent a problem-solving process distinct from the traditional sequential method. This graphical approach emphasizes understanding the problem and planning the solution in a structured and coherent manner. Additionally, the graphical representation of data serves as a tool to help students comprehend word problems more effectively, enhancing their understanding of mathematical concepts and improving their problem-solving efficiency. However, the study revealed that some groups of students were able to complete the answers in each section but struggled with the final section, which involved summarizing the results and providing justifications for their answers. This suggests that while the tool may be beneficial for low- or middle-ability students in solving mathematical problems, further refinement may be needed to address challenges in the reasoning and summarization stage.

3.2.2.3 The integration of the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers

The study of the DAPIC Problem-Solving Process reveals that students can effectively solve mathematical problems using this method. However, certain groups of students face challenges in correctly defining the problem and articulating their understanding of the given problem scenario. Without enough foundational knowledge, students struggle to initiate the problem-solving process and progress through the required mathematical steps (Chantarawong & Pruekpramool, 2020). Furthermore, it was observed that some

students were unable to adequately express their understanding during the Define step, failing to identify what they already knew from the provided problem context (Wijiya, 2020). The Four Corners and Diamond Graphic Organizers is a tool that fosters mathematical problem-solving skills by utilizing graphic organizers. While it is effective, it is not the sole process for improving problem-solving abilities. Certain student groups can complete all sections of the organizer but still struggle with the final part, which involves summarizing results and providing justifications for their answers. This suggests that the organizer may serve as a helpful tool for supporting students with low or average problem-solving abilities (Statema, 2012).

Therefore, the DAPIC Problem-Solving Process has been integrated with the Four Corners and Diamond Graphic Organizers. This integration aims to bridge the incomplete steps of both approaches, providing a comprehensive framework for students to solve mathematical problems more effectively. The integrated approach is summarized in Table 2.

Table 2 Integrating the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers

The DAPIC Problem-Solving Process	The Four Corners and Diamond Graphic Organizers	Integrating the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers
Step 1 Define: Understand the problem by identifying what needs to be determined, what is known, and what is required to solve the problem.	Part 1 Main Idea: Rewrite what the problem is asking for in your own words.	Step 1 Define This step integrates Step 1 (Define) from the DAPIC Problem-Solving Process with Part 1 (Main Idea) and Part 2 (Connections) from the Four Corners and Diamond Graphic Organizers, which is a step toward understanding the problem. In this step, students specify what the problem seeks to find and articulate the requirements of the problem in their own words (Main Idea). Additionally, they consider whether the information given in the problem is connected to their prior experiences in solving similar problems (Connections).
	Part 2 Connections: Identify and write down key information from the problem. Relate it to real-life scenarios and consider whether the students have encountered or solved a similar situation before.	
Step 2 Assess: Perform a problem-solving assessment by gathering relevant information or knowledge needed to solve the problem. This step involves connecting the collected information to the problem's requirements.		Step 2 Assess Step 2 involves evaluating the problem-solving process by identifying which information or mathematical knowledge should be selected for solving the problem. Students are encouraged to determine the reasons that will guide them in solving the problem before proceeding to plan a solution.
Step 3 Plan: Develop a plan or strategy to solve the problem. This includes identifying appropriate methods based on the nature of the problem and organizing the steps sequentially.	Part 3 Brainstorm: Develop and write potential strategies or mathematical methods for solving the problem.	Step 3 Plan This step integrates Step 3 (Plan) of the DAPIC Problem- Solving Process with Part 3 (Brainstorm) of the Four Corners and Diamond Graphic Organizers. It focuses on brainstorming strategies for solving the problem (brainstorm), where students write down the sequence of steps they plan to follow to arrive at a solution.

Table 2 Integrating the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers (Cont'd)

The DAPIC Problem-Solving Process	The Four Corners and Diamond Graphic Organizers	Integrating the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers
Step 4 Implement: Execute the planned solution to arrive at an answer. This step involves applying mathematical knowledge, principles, or concepts while also refining the action plan as necessary to address the problem effectively.	Part 4 Solve: Perform the necessary calculations to solve the problem, following the steps outlined during brainstorming.	Step 4 Implement: This step integrates Step 4 (Implement) of the DAPIC Problem-Solving Process with Part 4 (Solve) of the Four Corners and Diamond Graphic Organizers. It involves executing the planned solution by applying relevant mathematical knowledge, principles, and concepts. Students write out the calculations according to the steps outlined in the plan (Solve).
Step 5 Communicate: Analyze and conclude the results. This step includes presenting the findings to others, exchanging insights, and evaluating the results to ensure accuracy and consistency, often through written worksheets.	Part 5 Write: Summarize the solution step by step, explaining the reasoning behind each step to justify the answer.	Step 5 Communicate, Exchange, and Summary This step integrates Step 5 (Communicate) of the DAPIC Problem-Solving Process with Part 5 (Write) of the Four Corners and Diamond Graphic Organizers. It involves writing a summary of the results based on the steps followed in solving the problem, along with explaining the reasoning behind the solution (Write). In addition, students share their results and exchange insights with others.

From Table 2, the integration of the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers can be summarized as follows:

Step 1 Define: This step integrates Step 1 (Define) from the DAPIC Problem-Solving Process with Part 1 (Main Idea) and Part 2 (Connections) from the Four Corners and Diamond Graphic Organizers, which is a step toward understanding the problem. In this step, students specify what the problem seeks to find and articulate the requirements of the problem in their own words (Main Idea). Additionally, they consider whether the information given in the problem is connected to their prior experiences in solving similar problems (Connections).

Step 2 Assess: Step 2 involves evaluating the problem-solving process by identifying which information or mathematical knowledge should be selected for solving the problem. Students are encouraged to determine the reasons that will guide them in solving the problem before proceeding to plan a solution.

Step 3 Plan: This step integrates Step 3 (Plan) of the DAPIC Problem-Solving Process with Part 3 (Brainstorm) of the Four Corners and Diamond Graphic Organizers. It focuses on brainstorming strategies for solving the problem (Brainstorm), where students write down the sequence of steps they plan to follow to arrive at a solution.

Step 4 Implement: This step integrates Step 4 (Implement) of the DAPIC Problem-Solving Process with Part 4 (Solve) of the Four Corners and Diamond Graphic Organizers. It involves executing the planned solution by applying relevant mathematical knowledge, principles, and concepts. Students write out the calculations according to the steps outlined in the plan (Solve).

Step 5 Communicate, Exchange, and Summary: This step integrates Step 5 (Communicate) of the DAPIC Problem-Solving Process with Part 5 (Write) of the Four Corners and Diamond Graphic Organizers. It involves writing a summary of the results based on the steps followed in solving the problem, along with explaining the reasoning behind the solution (Write). In addition, students share their results and exchange insights with others.

The integration of the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers serves as a tool to assist students in mathematical problem-solving. It focuses on understanding the problem by identifying what the problem requires and connecting it to mathematical knowledge. It also involves assessing the problem to identify what information or mathematical knowledge will be used to solve it. Additionally, the process includes planning, implementing, and summarizing the results, along with the rationale used to find the answers. This aims to help students develop better mathematical problem-solving skills.

4. Results and Discussion

The assessment results of the lesson plans, assessed by experts, utilized a quality assessment form for each lesson plan, with a total score of 125 points. The quality assessment employed a Likert scale with the following ranges: "Excellent" average score between 4.50 and 5.00, "Good" average score between 3.50–4.49, "Neutral" average score between 2.50 and 3.49, "Fair" average score between 1.50–2.49, and "Needs Improvement" average score between 0–1.49. Overall, the four lesson plans received an average rating of "Excellent," which were assessed by experts, are presented in Table 3.

Table 3 The results of the assessment conducted on all lesson plans

Assessment List	Quality level			Statistic	
	The expert 1	The expert 2	The expert 3	$\bar{x}$	S.D.
Lesson Plan 1: The Elementary Counting Technique (Addition Principle)					
Total	125	120	111	118.67	8.49
Mean	5	4.8	4.44	4.75	0.34
Lesson Plan 2: The Elementary Counting Technique (Multiplication Principle)					
Total	125	122	111	119.33	7.07
Mean	5	4.88	4.44	4.77	0.28
Lesson Plan 3: Linear Permutations of Distinct Objects					
Total	125	123	111	119.67	7.07
Mean	5	4.92	4.44	4.79	0.28
Lesson Plan 4: Linear Permutations of Indistinct Objects					
Total	125	123	111	119.67	7.54
Mean	5	4.92	4.44	4.79	0.30

Each section of the lesson plans includes activity sheets and exercises designed around problem scenarios that align with the principles of the Elementary Counting Techniques and Linear Permutations. These materials aim to enhance students' analytical thinking and problem-solving skills within the framework of mathematical principles. This lesson plan integrates tools from the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers. The lesson plans are divided into four lesson plans as follows:

1) The Elementary Counting Technique
    1.1) Addition Principle
    1.2) Multiplication Principle
2) Linear Permutation
    2.1) Linear Permutations of Distinct Objects
    2.2) Linear Permutations of Indistinct Objects

The instructional tool developed by integrating the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers demonstrates consistency with Bloom's taxonomy by emphasizing the development of hierarchical cognitive skills. Bloom's framework divides learning objectives into six levels: Remember, Understand, Apply, Analyze, Evaluate, and Create (Anderson & Krathwohl, 2001). These

levels are effectively integrated into the teaching process. Follow The four lesson plans received an average "Excellent" rating on a Likert scale following expert assessment.

All lesson plans were revised to ensure consistency across objectives. Experts emphasized the use of consistent mathematical vocabulary across all plans to avoid confusion. All plans were thoroughly checked for accuracy before being used with students. Additionally, experts have highlighted the key points of all 4 lesson plans as follows:

1) When conducting learning activities, the instructor may provide guidance on recording details in the lesson plan to enhance clarity.

2) Consider writing learning objectives that combine knowledge and skills in the lesson plans more explicitly.

3) It is essential to review for typographical errors or missing words before implementing the lesson plans in practice.

The DAPIC Problem-Solving Process is a flexible framework that allows steps to be skipped or revisited as needed, depending on the nature of the problem. This approach supports mathematical problem-solving, aligning with research by Thongchoi et al. (2024), which indicates that the DAPIC-based instructional method enhances students' problem-solving abilities and cognitive processes. The lesson plan design emphasizes real-world problem contexts, allowing students to apply their knowledge effectively. This aligns with the findings of Risnanosanti and Ristontowi (2019), who suggest that the DAPIC Problem-Solving Process enables students to plan and solve problems efficiently. By connecting problem-solving to real-life situations, students can leverage prior knowledge and experiences, leading to improved analytical and problem-solving outcomes (Lampai, 2019). Additionally, Khakiki and Amir (2023) found that applying the DAPIC Problem-Solving Process in real-life scenarios enhances students' mathematical problem-solving skills. Wulandari, Roshmad, and Sugianto (2020) further explains that the DAPIC Problem-Solving Process framework enables students to define problems, assess situations, develop solution strategies, and implement their plans effectively. However, Wijiya (2020) noted that students sometimes struggle with the Define stage, particularly in identifying key problem requirements and available information. This difficulty, as observed by Simuang and Jantakoon (2022), often results in students spending more time in the Define stage than in other stages.

The Four Corners and Diamond Graphic Organizers are visual tools designed to enhance problem comprehension and facilitate structured problem-solving. These tools improve students' understanding of mathematical concepts and problem-solving efficiency, aligning with the research of Mei and Tengah (2021), which emphasizes their role in developing diverse learning skills. Furthermore, Gunasegar and Maat (2022) suggest that these graphic organizers provide an effective alternative strategy, allowing students to approach problem-solving flexibly from different starting points. By systematically recording and reviewing their problem-solving process, students can strengthen their comprehension and memory retention. Blessing and Taiwo (2024) support this, stating that graphic organizers improve mathematics learning by connecting new information to prior knowledge. Additionally, Obiukwu (2021) explains that these tools help learners clarify concepts, summarize reasoning, and create visual representations of problems, fostering deeper connections between mathematical knowledge and given problems. They also serve as diagnostic tools for teachers to identify misconceptions or errors in students' understanding. Sai et al. (2018) emphasize that breaking problems into smaller components through visual representation enhances problem comprehension and conceptual understanding.

While the DAPIC Problem-Solving Process is flexible, relying solely on it may not be enough, as challenges in the Define stage can hinder students' ability to fully identify problems. Integrating the Four Corners and Diamond Graphic Organizers enhances the DAPIC Problem-Solving Process by providing structured support in problem comprehension and solution planning. These tools help students break down problems into manageable parts and systematically record relevant information. By integrating the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers, this instructional approach aims to create a more comprehensive and effective problem-solving framework, ensuring that students can systematically analyze and solve mathematical problems with greater accuracy.

Based on expert recommendations, providing guidance on recording details in activity sheets is essential for instructors. This practice enhances understanding and clarity in each step of the DAPIC Problem-Solving Process and the use of graphic organizers. It also allows instructors to closely monitor and assess students' progress. Additionally, offering guidance on written documentation enables instructors to provide feedback, helping students identify and correct their errors, leading to improvements in their writing. This aligns with the research by Wongchai (2021), which states that writing is a crucial communication skill that serves as a medium for conveying messages to readers with accuracy, clarity, and completeness. Writing becomes more effective when practiced regularly, and receiving guidance and feedback further helps students review, refine, and improve the quality of their writing. Clearly defining learning objectives that integrate both knowledge and skills in lesson plans is crucial for fostering meaningful and effective learning. Specifying learning objectives or expected learning outcomes establishes clear expectations regarding students' anticipated behaviors. Moreover, well-defined objectives enable instructors to design learning activities and assessments that align precisely with the intended goals. Integrating knowledge and skills into learning objectives also promotes an interdisciplinary learning approach and the development of essential 21st-century skills. This aligns with the research of Chaowakeratipong (2017), which states that effective teaching and learning management stem from thorough lesson planning before instruction, including assessment strategies aligned with the set objectives. The process of defining learning objectives establishes the expected student outcomes at the end of the lesson, encompassing knowledge, procedural skills, and attitudes. These objectives can be written in two forms: general objectives, which describe broad learner characteristics, and specific or behavioral objectives, which outline precise, observable student behaviors.

Ensuring that lesson plans are free from writing errors or missing words is crucial for maintaining linguistic clarity. Errors in instructional materials can lead to confusion, misinterpretation, and a reduction in the credibility of the lesson content. This highlights the importance of thoroughly reviewing lesson plans for errors before implementation. Moreover, such errors may contribute to increased teacher stress. This aligns with Keyser (2019), who explained that despite the availability of proofreading tools, such as spell checkers, errors can still go unnoticed. These mistakes may not only impact teachers' stress levels but also reduce the effectiveness of instruction. To prevent such issues, emphasis should be placed on the proofreading process and careful review of lesson plans before they are put into practice.

5. Conclusion

In this paper, the researcher designed and developed lesson plans by integrating the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers. The study included four lesson plans: 1) the Elementary Counting Technique (Addition Principle), 2) the Elementary Counting Technique (Multiplication Principle), 3) Linear Permutations of Distinct Objects, and 4) Linear Permutations of Indistinct Objects.

Each lesson plan emphasizes the creation of diverse problem situations that align with the content of the plan. Integration the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers, which consists of five steps: Step 1 Define, Step 2 Assess, Step 3 Plan, Step 4 Implement, and Step 5 Communicate, Exchange, and Summary. The integration using the DAPIC Problem-Solving Process with the Four Corners and Diamond Graphic Organizers will be a tool to help promote and develop mathematical problem-solving skills for students. These lesson plans were refined and improved based on expert recommendations to enhance the mathematical problem-solving skills of high school students. Overall, the four lesson plans received an average rating of "Excellent" on a Likert scale.

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