Calculations of Section Capacity of an Irregular Shape Reinforced Concrete Column Using Excel and VBA

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Abstract

A Visual Basic for Applications (VBA) workbook was developed to design reinforced concrete columns in any shape, including irregular shapes. A design procedure was employed where equivalent rectangular stress block, as typically used in the ACI building code, was assumed for concrete stress at the ultimate state. The maximum strain in extreme concrete fiber is equal to 0.003. Tension in concrete is neglected. The stress-strain relationship in steel reinforcement is elastic-perfectly plastic. Integrals were required to obtain the area and the first moments in the procedure. The task was difficult for irregular column cross sections, and Green's theorem was used to simplify this task. The procedure was transformed into equations in Excel sheets and VBA subroutines. To use the workbook, the shape of reinforced concrete column and reinforcement data are input. The plastic centroid (PC) of the section is then calculated by the workbook. Distances from the applied load to PC are then defined. The location and orientation of the Neutral Axis (NA) of the section are assumed. The axial force of the section and its eccentricities are calculated under equilibrium conditions. VBA will iterate the process until the computed eccentricities conform with the required eccentricities.

The abilities of the developed workbook were demonstrated with five examples of columns with different shapes, including irregular ones. The results from the workbook for circular, L-shaped, hollow-square, open-box, and T-shaped columns were consistent with those of a well-known program. The workbook can iterate to get results within a second. It has been confirmed that the workbook can be used with columns from any section.

Keywords: *Reinforced Concrete Column, Irregular Column, Excel, VBA.*

1. Introduction

Columns are used to transfer forces from building floors to foundations. Internal forces in a column are axial force, bending moments, and shear forces. The primary objective of column design is to resist the combined effects of axial force and bending moments. Excel equations can be used to design simple shape columns, such as circular and rectangular shape columns, without difficulty. The design of irregular shaped columns, however, requires considerable time and effort. Numerous publications present the concept of analyzing reinforced concrete (RC) sections with axial load and biaxial bending. Warner, Rangan, Hall, & Faulkes (1998) clearly showed the procedure for section analysis which can be used for further developments.

Barzegar and Erasito (1995) developed worksheets to analyze RC short columns subjected to combined axial compression and biaxial bending. They provided information about their development which can be utilized for other developments. Green theorem was used for the computation of section properties. They assumed a rectangular stress block to compute concrete stresses. The performance of their development was demonstrated through various examples. Rodriguez and Aristizabal-Ochoa (1999) presented a general method that determines the biaxial interaction diagrams for any orientation of the neutral axis of a RC short column of any cross section under axial load and bending about two axes. The ultimate strength of the section was expressed in closed form. A nonlinear stress-strain relationship was employed for the concrete while a multilinear elastoplastic relationship was used for the reinforcement The proposed method can be reproduced by another user with minimal calculations.

Papanikolaou (2012) presented a new methodology for the ultimate strength and moment–curvature analysis of arbitrary composite sections under biaxial bending and axial load. The performance of the methodology was demonstrated through various case studies, comparisons, and benchmarks. In terms of

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stability and speed, a very satisfactory performance   was obtained. The method can provide a solid basis for further developments. Al-Ansari and Afzal (2019) proposed a simplified method for analyzing irregular column sections. There are eight irregular column section types which are   equivalent to square sections. The equivalent square sections were tested with commercial software, and the results were in agreement. However, the method can be applied to eight types of sections and the results are only approximate.

It is interesting to use the knowledge from these papers to develop a low-cost program for the design of irregular-shaped reinforced concrete columns. The program will be based on Excel and its embedded Visual Basic for Applications (VBA), which is employed by engineers in their everyday work.

2. Objectives

This research has two main objectives which are as follows:
1) To obtain a procedure for designing columns in any shape, including irregular shapes.
2) To develop an Excel workbook to design irregular- shaped columns using the obtained procedure.

3. Methods

The procedure for designing irregular shaped reinforced concrete columns was summarized. The procedure was applied in an Excel workbook. VBA was employed to enhance the capabilities of Excel.

3.1 Design Procedure

The concepts presented in Warner et al. (1998) and Barzegar and Erasito (1995) can be used to design a reinforced concrete column of any configuration subjected to eccentric compression (N), as shown in Figure 1. Equilibrium conditions for the column can be expressed as follows:

$$N = \iint_{A_c} f_c(X,Y)dXdY + \sum_{j=1}^{m} f_{sj}(X_j, Y_j)A_{sj} \tag{1}$$

$$M_X = - \iint_{A_c} Y f_c(X,Y)dXdY - \sum_{j=1}^{m} Y_j f_{sj}(X_j, Y_j)A_{sj} \tag{2}$$

$$M_Y = \iint_{A_c} X f_c(X,Y)dXdY + \sum_{j=1}^{m} X_j f_{sj}(X_j, Y_j)A_{sj} \tag{3}$$

where $f_c(X, Y)$ and A_c represent the stress function and area of concrete, respectively. Compressive stress is positive, whereas tensile stress is negative. $f_{sj}(X_j, Yj)$ and A_{sj} represent the stress and area of steel reinforcement j, respectively, while m is the total number of reinforcements in the section.

At ultimate limit state, the distribution of concrete stress is assumed to be an equivalent rectangular stress block of magnitude 0.85fc', as typically used in the building code of the American Concrete Institute (ACI). The maximum strain in extreme concrete fiber is equal to 0.003. Tension in concrete is neglected. The stress-strain relationship in steel reinforcement is elastic-perfectly plastic. The behavior of steel reinforcement in tension and compression is the same. Equations 1 to 3 can be rewritten for the ultimate forces (N_u, M_{uX} and M_{uY}) as follows:

$$N_u = 0.85f'_c \iint_{A_{ec}} dXdY + \sum_{j=1}^{m} f_{sj}A_{sj} \tag{4}$$

$$M_{uX} = -0.85f'_c \iint_{A_{ec}} YdXdY - \sum_{j=1}^{m} Y_j f_{sj}A_{sj} \tag{5}$$

$$M_{uY} = 0.85f'_c \iint_{A_{ec}} XdXdY + \sum_{j=1}^{m} X_j f_{sj}A_{sj} \tag{6}$$

where A_{ec} represents the effective concrete area under compression. The expression for the steel stress (f_{sj}) is as follows:

$$f_{sj} = E_s \varepsilon_{sj} \qquad \text{when} \qquad \varepsilon_{sj} < \varepsilon_{sy} \text{ and} \tag{7}$$
$$f_{sj} = f_{sy} \qquad \text{when} \qquad \varepsilon_{sj} \geq \varepsilon_{sy} \tag{8}$$

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where ε_{sj} is the strain at the location of bar j and E_s and f_{sy} are the modulus and yield stress of steel reinforcement, respectively.

Equations 4 to 6 show that integrals are required to obtain area and first moments. The task is difficult for irregular column cross sections. Green's theorem (Folie, 1971) is used to simplify the task. The area (A) and its first moments ($A\overline{X}$ and $A\overline{Y}$) can be obtained as follows:

$$A = \frac{1}{2} \sum_{i=1}^{n} a_i \tag{9}$$
$$A\overline{Y} = \frac{1}{6} \sum_{i=1}^{n} a_i(Y_i + Y_{i+1}) \tag{10}$$
$$A\overline{X} = \frac{1}{6} \sum_{i=1}^{n} a_i(X_i + X_{i+1}) \tag{11}$$

where $a_i = X_i Y_{i+1} - X_{i+1} Y_i$ and n is the number of points defining the boundaries of the column section. The points should be numbered counterclockwise on the exterior boundaries and clockwise on the interior boundaries (Figure 2). $\overline{X}$ and $\overline{Y}$ are the coordinates of the centroid of the area.
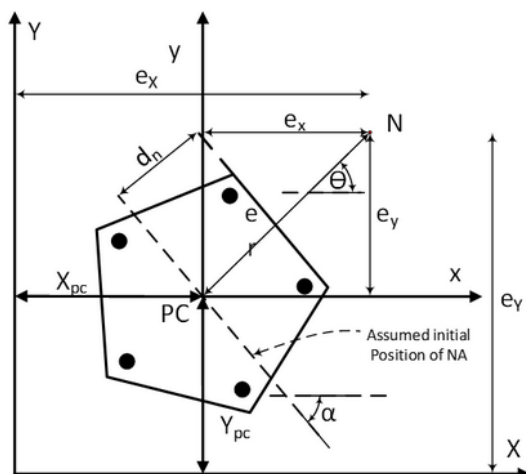
Figure 1: Column with Eccentric Loading
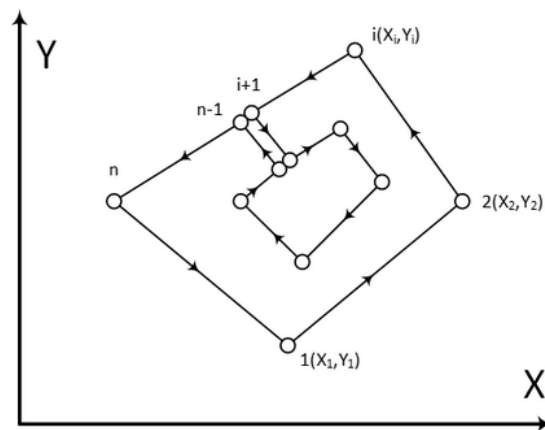
Figure 2: Numbering of Points along the Section Boundaries

Eccentricities of load on the section are measured from the plastic centroid of the section (PC). The location of PC (X_{PC} and Y_{PC}) can be obtained from the condition where a uniform compressive stress of 0.85fc' exists throughout the section and all steel reinforcements are under yield compression.

$$X_{PC} = \frac{0.85f_c'A\overline{X} + \sum_{j=1}^{m} X_j f_{sj} A_{sj}}{0.85f_c'A + \sum_{j=1}^{m} f_{sj} A_{sj}} \tag{12}$$

$$Y_{PC} = \frac{0.85f_c'A\overline{Y} + \sum_{j=1}^{m} Y_j f_{sj} A_{sj}}{0.85f_c'A + \sum_{j=1}^{m} f_{sj} A_{sj}} \tag{13}$$

Following the explanation above, the procedure for the design of irregular-shaped reinforced concrete columns can be summarized as follows (refer to Figure 1):
a) Input the shape of the reinforced concrete column and reinforcement data.
b) Calculate the plastic centroid (PC) of the section.
c) Specify distances from the applied load to PC (e_x and e_y).
d) Assume the location (d_n) and orientation (α) of the Neutral Axis (NA) of the section.
e) Calculate the internal forces of the section (N_u, M_{uX} and M_{uY}) using Equations 4 to 6.

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f) Calculate the eccentricities of internal forces from Step e and Parameter Δ_r.

$$e_{nX} = \frac{M_{uY}}{N_u} \qquad \text{and} \qquad e_{nY} = \frac{M_{uX}}{N_u} \qquad (14)$$

$$\Delta_r = \sqrt{(e_{nX} - e_X)^2 + (e_{nY} - e_Y)^2} \qquad (15)$$

g) If the calculation error (Δ_r) is not negligible, adjust the neutral axis (d_n and α in Step e) and redo Steps e and f. The design is complete when the error in Equation 15 is nearly zero.

3.2 Program Development

A workbook with VBA was developed to design RC columns with any shapes, including irregular shapes. The procedure in Section 3.1 was employed. The workbook is composed of 3 worksheets labeled "input", "program" and "figure".

Worksheet "input" (Figure 3) contains all the input data, such as the cross-section boundary coordinates, steel yield stress, total number and position of reinforcements, the size of rebar, and the compressive strength of concrete. This program was designed to input up to 100 reinforcing bars and 100 boundary points.

Boundary ID	mm X	mm Y		Reinforce ID	mm Xi	mm Yi		DB	16	
1	200	0		1	0	160		fc' =	32MPa	
2	184.776	76.5367		2	94.0456	129.443		fy =	400MPa	
3	141.421	141.421		3	152.169	49.4427		Es =	200000MPa	
4	76.5367	184.776		4	152.169	-49.4427				
5	-0.1317	200.105		5	94.0456	-129.443				
6	-76.5367	184.776		6	2E-14	-160				
7	-141.421	141.421		7	-94.0456	-129.443				
8	-184.776	76.1853		8	-152.169	-49.4427				
9	-200	0		9	-152.169	49.4427				
10	-184.776	-76.5367		10	-94.0456	129.443				

Figure 3: Overview of the Worksheet "input"

Worksheet "program" reads data from the first worksheet and arranges the data in a closed boundary format for further analysis of ultimate load and moments. Both the number of boundary points and the number of reinforcements are limited to 100. Important calculations are performed in this worksheet. The worksheet was divided into three parts, i.e., Parts A, B and C (Figure 4).

Part A summarized input and output parameters (Figure 5). Important input parameters are concrete strength (fc'), maximum strain at extreme concrete fiber (ε_u), yield stress of reinforcement (fy), and elastic modulus of the reinforcement (Es). From these inputs, parameters defining the equivalent rectangular stress block, such as the ratio of the stress block depth (β) and the depth of the stress block (a) are calculated following the ACI Code. Other input parameters are distances from the applied load to PC (eX and eY). Trial parameters, such as the maximum distance from the farthest boundary point to the neutral axis (Cmax) and the neutral axis inclination angle (α) shall be input in this part. The right hand side area of Part A shows output results, i.e., ultimate axial force (Nu), ultimate moment around the X axis (MuY), and ultimate moment around the Y axis (MuX). Eccentricities from these moments are calculated as enX and enY and checked with the required values eX and eY. The program will run until the errors between these eccentricities are within an acceptable limit.

Part B (Figure 6) consists of calculations regarding stresses in concrete. The columns titled "Area", "A-X" and "A-Y" are calculations of area and moments of area to obtain the plastic centroid (Xpc and Ypc). In the "Di" column, the distance from a boundary point to a line parallel to the trial neutral axis (angle = α) and passing to the plastic centroid is calculated. The equation for "Di" is as follows:

$$D_i = \frac{Y - X\tan(\alpha) - Ypc + Xpc\tan(\alpha)}{\sqrt{\tan^2\alpha + 1}} \qquad (16)$$

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The distance from the boundary point to the trial neutral axis is calculated in the next column (labeled "Ci"). The distance from the boundary point to the edge of the rectangular stress block in the column labeled "Ci – a0" whereas a0 is the distance between the trial neutral axis and the edge of the rectangular stress block (Figure 7). Coordinates of compressive area are calculated in columns labeled "Xcom" and "Ycom", using the flow chart in Figure 8. Then, the area and moments of area of the compressive area are calculated in the columns labeled "A, com", "A, com·Xcom" and "A, com·Ycom", respectively. Finally, the total area and moments of area of the compressive area are multiplied with the height of the rectangular stress block (0.85 fc') to get the ultimate values of the internal forces of the column section (Nu, MuX, and MuY).
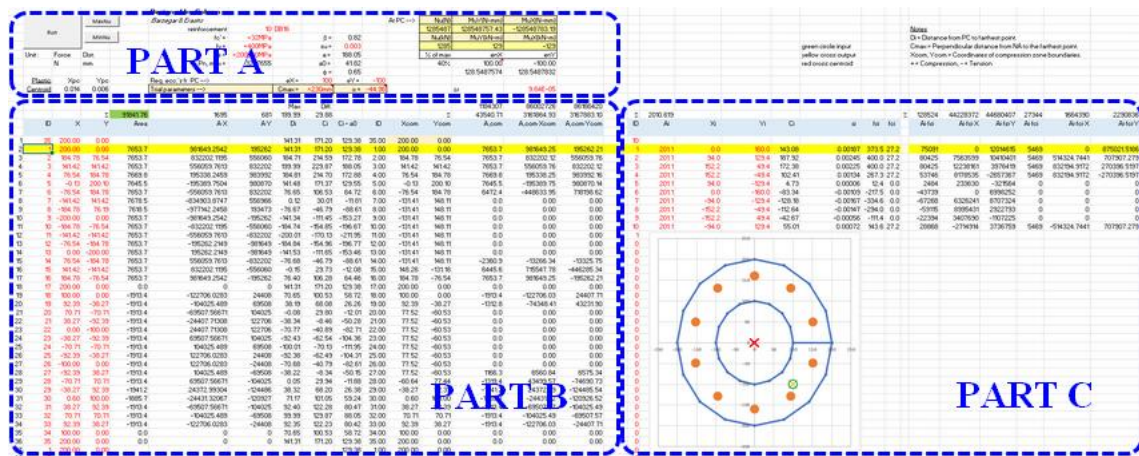
Figure 4: Overview of the Worksheet "program"
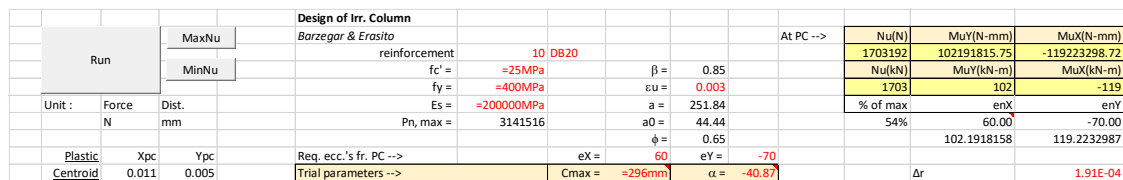
Figure 5: Part A of the Worksheet "program"

Figure 6: Part B of the Worksheet "program"

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Part C (Figure 9) consists of calculations regarding stresses in reinforcements. Similar procedures in Part B are used. Column "Ai" shows the area of each reinforcement while Columns "Xi" and "Yi" show the coordinates of the reinforcement. "Ci" is the distance from reinforcement to the trial neutral axis and can be calculated as follows:

$$Ci = \frac{Yi - Xi\tan(\alpha) - Ypc + Xpc\tan(\alpha)}{\sqrt{\tan^2\alpha + 1}} + Cmax - Dmax \qquad (17)$$

Cmax and Dmax are the distances from the farthest boundary point to a line parallel to the trial neutral axis and a line parallel to the trial neutral axis and passing to the plastic centroid, respectively. The ε_i is the strain in the reinforcement. The fsi and fci are tensile and compressive stresses in the reinforcement which follow elastic-perfectly plastic behavior. Forces and moments around the Y and X axes are in the columns labeled "Ai·fsi", "Ai·fsi·X" and "Ai·fsi·Y", respectively. The forces and moments in the overlapping area with concrete (Ai·fci, Ai·fci-X, and Ai·fci-Y) are calculated and deducted from the concrete forces and moments. A figure with the shape of a column and reinforcements is also in Part C. This is for checking the correctness of input.
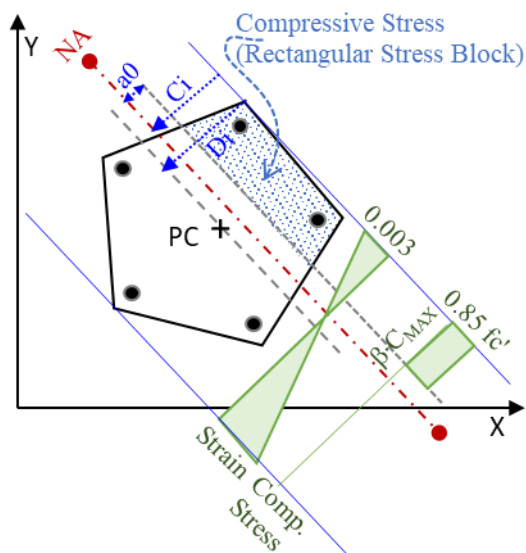
Figure 7: Parameters defining the Neutral Axis and Stress Block
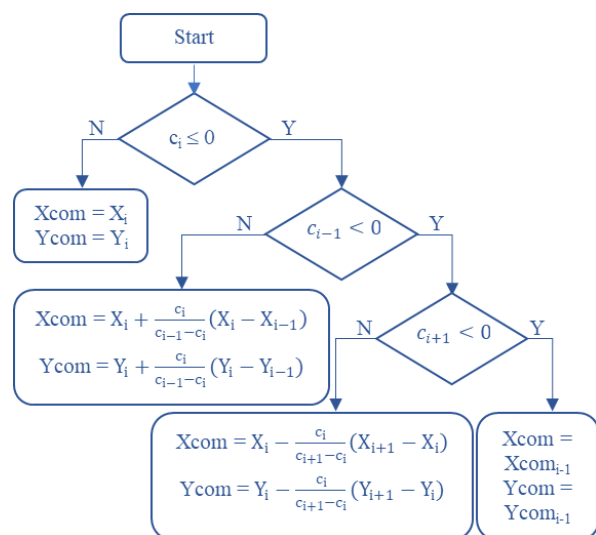
Figure 8: A Flow Chart to Calculate Xcom and Ycom

Figure 9: Part C of the Worksheet "program"

There are three main VBA subroutines (Figure 10) with three corresponding buttons (Figure 5) in the worksheet "program" to call them. Subroutine "RUN" will execute Solver, which is a Microsoft Excel add-in program used for what-if analysis, to adjust for appropriate values of Cmax and α in order to minimize

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the error Δ. Subroutine "MaxNu" uses Solver to adjust NA depth and angle of inclination to determine the maximum axial load the column can withstand. Oppositely, the subroutine "MinNu" determines the minimum axial load.

4. Results and Discussion

In this section, the ability to design reinforced concrete columns of any shape using the developed workbook was demonstrated. Five examples were investigated, i.e., circular column, an L-shaped column, a hollow-square column, an open-box column, and a T-shaped column. The worksheet with VBA can iterate to obtain results within a second. The results were compared to those of a well-known program, sPColumn by StructurePoint (2021). Details of the investigations were shown in the following.

4.1 Circular Column

In this section, simple shape column, known as a circular column with a radius of 200 mm reinforced by 10 DB28, was investigated. The concrete and steel strength were 32 MPa and 400 MPa, respectively. The column was analyzed with eccentricities, $e_x = 141$ mm and $e_y = -141$ mm. Input and results of the investigation were shown in Figure 11. Figure 12b showed that Nu, MuY, and MuX obtained from the workbook were consistent with those from sPColumn.

```
Sub RUN()
    SolverOk SetCell:="$O$10", MaxMinVal:=2, ValueOf:=0, ByChange:="$I$10:$I$11", _
        Engine:=1, EngineDesc:="GRG Nonlinear"
    SolverSolve
End Sub

Sub MaxNu()
    Range("M3").Select
    SolverOk SetCell:="$M$3", MaxMinVal:=1, ValueOf:=0, ByChange:="$I$10:$I$11", _
        Engine:=1, EngineDesc:="GRG Nonlinear"
    SolverSolve
End Sub

Sub MinNu()
    Range("M3").Select
    SolverOk SetCell:="$M$3", MaxMinVal:=2, ValueOf:=0, ByChange:="$I$10:$I$11", _
        Engine:=1, EngineDesc:="GRG Nonlinear"
    SolverSolve
End Sub
```

Figure 10 Main VBA Subroutines

Design of Irr. Column						At PC -->	Nu(N)	MuY(N-mm)	MuX(N-mm)
Barzegar & Erasito							1656617	233582581.60	-233583564.59
reinforcement	10	DB28					Nu(kN)	MuY(kN-m)	MuX(kN-m)
fc' =	=32MPa		β =	0.82			1657	234	-234
fy =	=400MPa		εu =	0.003			% of max	enX	enY
Es =	=200000MPa		a =	181.18			29%	141.00	-141.00
Pn, max =	5626401		a0 =	40.29					
			ϕ =	0.65					
Req. ecc.'s fr. PC -->			eX =	141	eY =	-141			
Trial parameters -->			Cmax =	=221mm	α =	-44.88		Δr	4.21E-04

Figure 11: Main Input and Output of the Workbook for the Circular Column Example
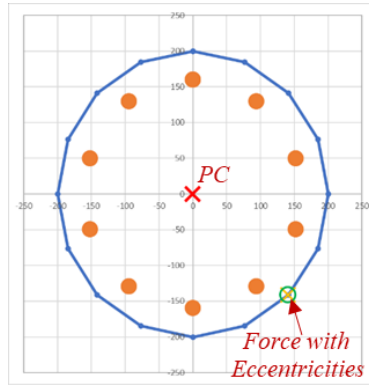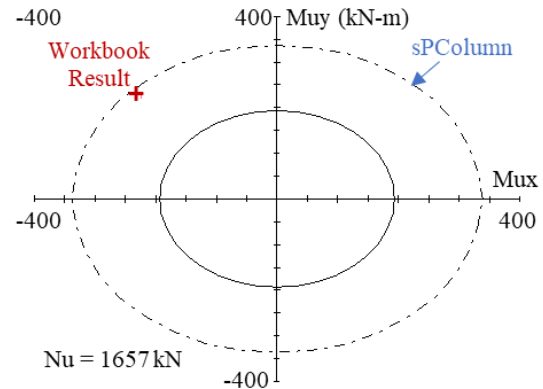
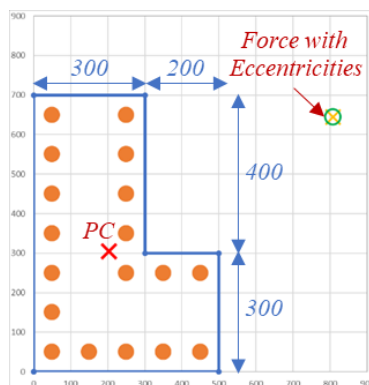a) Details of Column from Workbook   b) Comparison of Result with sPColumn

Figure 12: Comparison of Results in the Circular Column Example

4.2 L-Shaped Column

An L-shaped column with maximum dimensions of 500x700 mm and reinforced by 18 DB20 was investigated in this example. The concrete and steel strength were 25 MPa and 400 MPa, respectively. The column was analyzed with eccentricities, $e_x = 604$ mm and $e_y = 340$ mm. Input and results of the investigation were shown in Figure 13. Figure 14b showed that Nu, MuY and MuX obtained from the workbook were consistent with those from sPColumn. The results also showed the efficiency of the workbook to analyze such an irregular column with high eccentricities.

Design of Irr. Column						At PC -->	Nu(N)	MuY(N-mm)	MuX(N-mm)
Barzegar & Erasito							792037	478396846.64	269292375.79
reinforcement	18 DB20						Nu(kN)	MuY(kN-m)	MuX(kN-m)
fc' =	=25MPa		β =	0.85			792	478	269
fy =	=400MPa		εu =	0.003			% of max	enX	enY
Es =	=200000MPa		a =	208.56			10%	604.01	340.00
Pn, max =	7879281		a0 =	36.80					
			φ =	0.65					
Req. ecc.'s fr. PC -->			eX =	604	eY =	340			
Trial parameters -->			Cmax =	=245mm	α =	-89.99		Δr	8.01E-03

Figure 13: Main Input and Output of the Workbook for the L-Shape Column Example
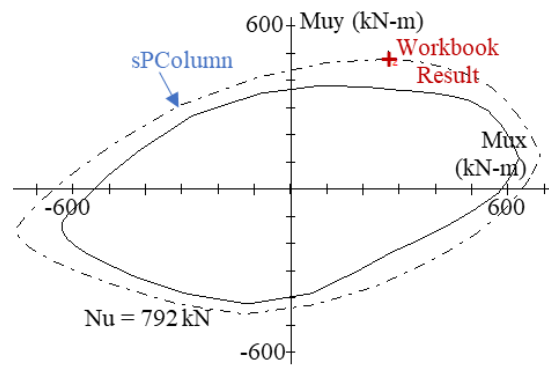
a) Details of Column from Workbook   b) Comparison of Result with sPColumn

Figure 14: Comparison of Results in the L-Shape Column Example

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4.3 Hollow-Square Column

In this example, a Hollow-Square column with 450x450 mm dimensions and a 150x150 mm internal void reinforced by 20 DB20 was investigated in this example. The concrete and steel strength were 28 MPa and 370 MPa, respectively. The column was analyzed with eccentricities, e_x = -300 mm and e_y = 100 mm. Input and results of the investigation were shown in Figure 15. Figure 16b showed that Nu, MuY, and MuX obtained from the workbook were consistent with those from sPColumn. The workbook can be used to analyze the hollow column in terms of combined tension and moments.

Design of Irr. Column						At PC -->	Nu(N)	MuY(N-mm)	MuX(N-mm)
Barzegar & Erasito							-878333	263499551.74	-87833046.44
reinforcement	20	DB20					Nu(kN)	MuY(kN-m)	MuX(kN-m)
fc' =	=28MPa		β =	0.85			-878	263	-88
fy =	=370MPa		εu =	0.003			% of max	enX	enY
Es =	=200000MPa		a =	104.47			-14%	-300.00	100.00
Pn, max =	6459239		a0 =	18.86					
			φ =	0.65					
Req. ecc.'s fr. PC -->			eX =	-300	eY =	100			
Trial parameters -->			Cmax =	=123mm	α =	-76.46		Δr	5.19E-04

Figure 15: Main Input and Output of the Workbook for the Hollow-Square Column Example

a) Details of Column from Workbook

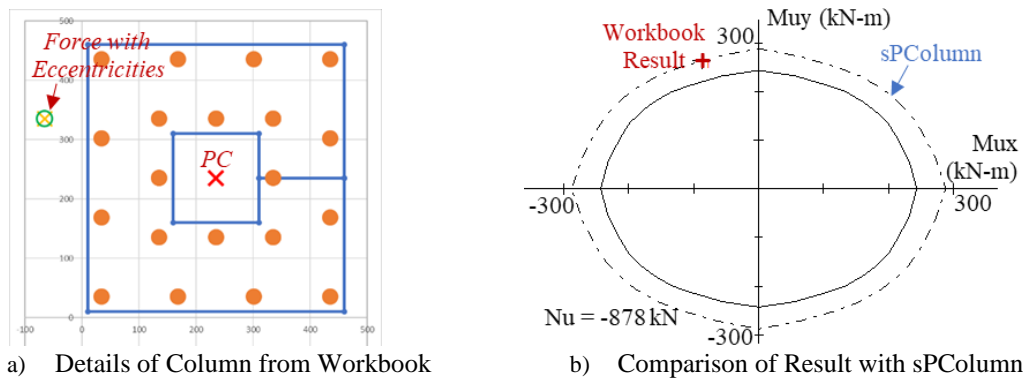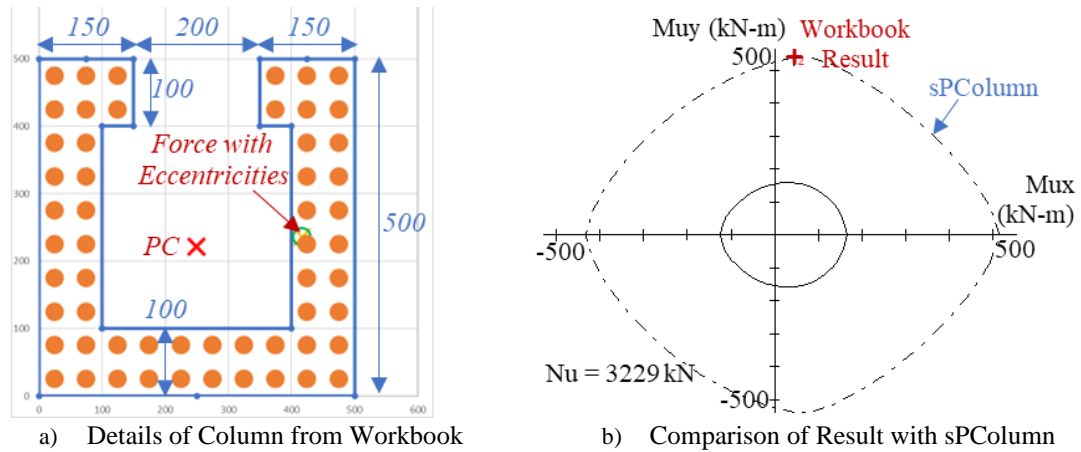b) Comparison of Result with sPColumn

Figure 16: Comparison of Results in the Hollow-Square Column Example

4.4 Open-Box Column

An Open-Box column with 500x500 mm dimensions and a 300x300 mm internal void reinforced by 56 DB14 was investigated in this example. The concrete and steel strength were 25 MPa and 400 MPa, respectively. The column was analyzed with eccentricities, e_x = 167 mm and e_y = 14 mm. Input and results of the investigation were shown in Figure 17. Figure 18b showed that Nu, MuY, and MuX obtained from the workbook were consistent with those from sPColumn. The results also show the efficiency of the workbook to analyze such a similar section to a lift core.

Design of Irr. Column						At PC -->	Nu(N)	MuY(N-mm)	MuX(N-mm)
Barzegar & Erasito							3229316	539296819.83	45210597.34
reinforcement	56	DB14					Nu(kN)	MuY(kN-m)	MuX(kN-m)
fc' =	=25MPa		β =	0.85			3229	539	45
fy =	=400MPa		εu =	0.003			% of max	enX	enY
Es =	=200000MPa		a =	350.65			52%	167.00	14.00
Pn, max =	6240026		a0 =	61.88					
			φ =	0.65					
Req. ecc.'s fr. PC -->			eX =	167	eY =	14			
Trial parameters -->			Cmax =	=413mm	α =	-89.94		Δr	3.37E-04

Figure 17: Main Input and Output of the Workbook for the Open-Box Column Example

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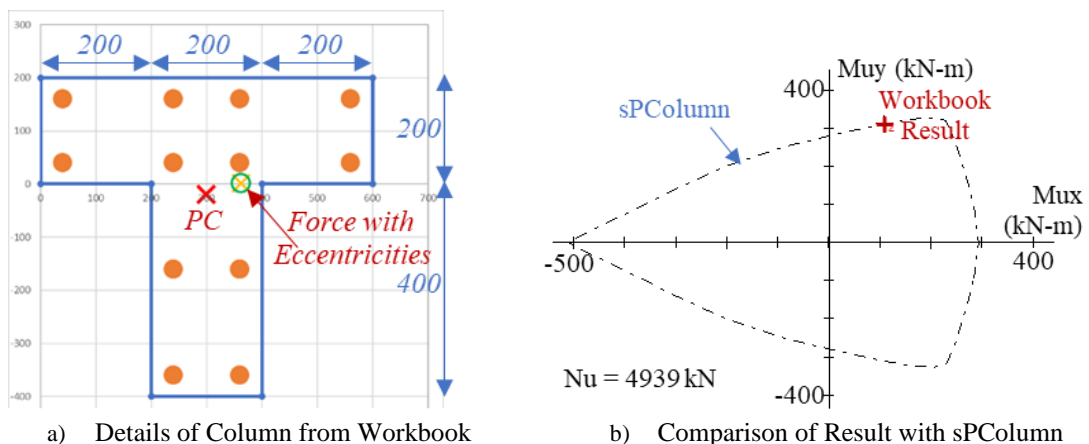a) Details of Column from Workbook   b) Comparison of Result with sPColumn

Figure 18: Comparison of Results in the Open-Box Column Example

4.5 T-Shaped Column

A T-Shaped column with 600x200 mm flange width and thickness and 400x200 mm web width and thickness reinforced by 12 DB28 was investigated in this example. The concrete and steel strength were 25 MPa and 370 MPa, respectively. The column was analyzed with eccentricities, e_x = 63 mm and e_y = 22 mm. Input and results of the investigation were shown in Figure 19. Figure 20b showed that Nu, MuY, and MuX obtained from the workbook were consistent with those from sPColumn.

Design of Irr. Column						At PC -->	Nu(N)	MuY(N-mm)	MuX(N-mm)
Barzegar & Erasito							4939190	311168249.80	108657334.80
reinforcement	12	DB28					Nu(kN)	MuY(kN-m)	MuX(kN-m)
fc' =	=25MPa		β =	0.85			4939	311	109
fy =	=370MPa		εu =	0.003			% of max	enX	enY
Es =	=200000MPa		a =	428.43			72%	63.00	22.00
Pn, max =	6826923		a0 =	75.60					
			φ =	0.65					
Req. ecc.'s fr. PC -->			eX =	63	eY =	22			
Trial parameters -->			Cmax =	=504mm	α =	-89.46		Δr	9.94E-04

Figure 19: Main Input and Output of the Workbook for the T-Shaped Column Example

a) Details of Column from Workbook   b) Comparison of Result with sPColumn

Figure 20: Comparison of Results in the T-Shaped Column Example

5. Conclusion

A workbook with VBA was developed to design reinforced concrete columns in any shape, including irregular shapes. An equivalent rectangular stress block was assumed for concrete stress at the ultimate state. Tension in concrete is neglected. The stress-strain relationship in steel reinforcement is elastic-perfectly plastic. Green's theorem was used to simplify an integral task. The procedure was transformed into equations in Excel sheets and VBA subroutines. Iteration is used until the computed eccentricities conform with the required eccentricities. There are limitations on the numbers of boundary points and reinforcements at 100. The abilities of the developed workbook were demonstrated with five examples of columns with different shapes, including irregular ones. The results from the workbook for circular, L-shaped, hollow-square, open-box, and T-shaped columns were consistent with those from a well-known program. The workbook can iterate to get results within a second. The workbook can find a set of results at a time. The Ability to automatically generate interaction diagrams should be added to the workbook. The open nature of the workbook can be utilized for further studies such as optimization of column sections and automatic generation of drawings from the design workbook.

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